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Rossby rip currents

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[1] Oceanic Rossby waves and eddies flux energy and fluid westward, the latter through the Stokes drift or bolus transport. While the wave energy is largely dissipated at the western boundary, mass conservation requires that the fluid be returned offshore through Rossby rip currents. The form and magnitude of these rip currents are investigated through linear Rossby wave theory, a nonlinear numerical model, and analysis of sea surface height satellite observations. The net eastward volume transport by Rossby rip currents over the global ocean is estimated to be of order 10 Sv (1 Sv $\equiv 10^6$ m^3 s^{-1}). In an eddying ocean, both the westward Stokes drift and eastward rip currents can assume the form of banded quasi-zonal jets. **Citation:** Marshall, D. P., B. Vogel, and X. Zhai (2013), Rossby rip currents, *Geophys. Res. Lett.*, *40*, 4333–4337, doi:10.1002/grl.50842.

1. Introduction

[2] As surface waves approach a beach, they transport water onshore through their Stokes drift. Mass conservation requires that this water be returned offshore, achieved through intense seaward jets known as *rip currents*. The fluid dynamics describing the structure and spacing of rip currents is complicated and involves a variety of mechanisms including wave radiation stresses, bathymetric variations, and instabilities of alongshore currents (for a review, see *Dalrymple et al.* [2011]).

[3] Less well appreciated is that westward propagating Rossby waves and eddies also transport fluid westward through a Stokes drift. By analogy with surface waves breaking at a beach, this water may be returned eastward through mean flows that we term *Rossby rip currents*.

[4] The aim of this short contribution is to investigate the cause, structure, and magnitude of Rossby rip currents in the ocean through the application of linear Rossby wave theory (section 2), numerical calculations with a nonlinear reduced-gravity model (section 3), and analysis of satellite observations of sea surface height variability (section 4). A brief concluding discussion is given in section 5.

2. Theoretical Background

2.1. Classical Rip Currents

[5] Consider linear shallow water waves in an ocean of depth H with sea surface elevation and velocity anomalies given by:

$$\eta' = \eta_0 \sin(kx - \omega t), \quad u' = u_0 \sin(kx - \omega t),$$

where k is the wave number, ω is the angular frequency, η_0 and u_0 are the wave amplitudes, x is distance, and t is time.

[6] Correlations between fluid parcel displacements and velocity gradients result in a rectified displacement of fluid parcels over a wave period, in the direction of wave propagation, known as *Stokes drift*. As shown by *Lee et al.* [1997], the Stokes drift velocity for linear waves is equivalent to the bolus velocity [*Gent et al.*, 1995],

$$c_s = \frac{\overline{\eta' u'}}{H} = \frac{\overline{\eta'^2}}{2H^2} c, \tag{1}$$

where $c = \sqrt{gH}$ is the nondispersive shallow water wave speed, g is the gravitational acceleration, and the overbar represents a time average.

[7] Now consider surface waves approaching a beach. While the wave energy is mostly dissipated at the beach, mass conservation requires that the onshore transport by the Stokes' drift is compensated by offshore Eulerian-mean currents (Figure 1). The fluid dynamics of how and why this return flow assumes the form of narrow rip currents within the wave surf zone is complicated [see *Dalrymple et al.*, 2011] but of secondary importance in the present context.

2.2. Rossby Rip Currents

[8] Westward propagating Rossby waves and eddies [e.g., *Chelton et al.*, 2007] flux energy westward. *Zhai et al.* [2010] use altimetric and hydrographic data to estimate the westward linear eddy energy flux associated with the first baroclinic mode:

$$F^{(x)} = \int \rho g_p h_0 \, \overline{h' u'} \, dy, \tag{2}$$

where ρ is density, g_p and h_0 are the reduced gravity and equivalent depth of the first baroclinic mode, h' and u' are the velocity and (effective) layer thickness anomalies, and x and y are the zonal and meridional coordinates. Poleward of 10° latitude, the global energy flux into the western boundary layers is approximately 0.1–0.2 TW (slightly larger if nonlinear terms are included in (2)). In idealized model calculations, *Zhai et al.* [2010] find that the vast majority of eddy energy is dissipated within the western boundary layers (the *Rossby graveyard*), just as the energy fluxed onshore by surface waves is dissipated near the shoreline.

[9] However, the linear energy flux is also proportional to the eddy bolus transport. Thus, concomitant with the westward linear energy flux is a westward Stokes drift. Taking $g_p = 1.5 \times 10^{-2}$ m s^{-2}, $h_0 = 750$ m, and $\rho_0 \sim 10^3$ kg m^{-3} (as in *Zhai et al.* [2010]) gives a global Stokes drift of approximately 10–20 Sv (1 Sv $\equiv$ 10^6 m^3 s^{-1}) into the western boundary layers. While the eddy energy can be dissipated,

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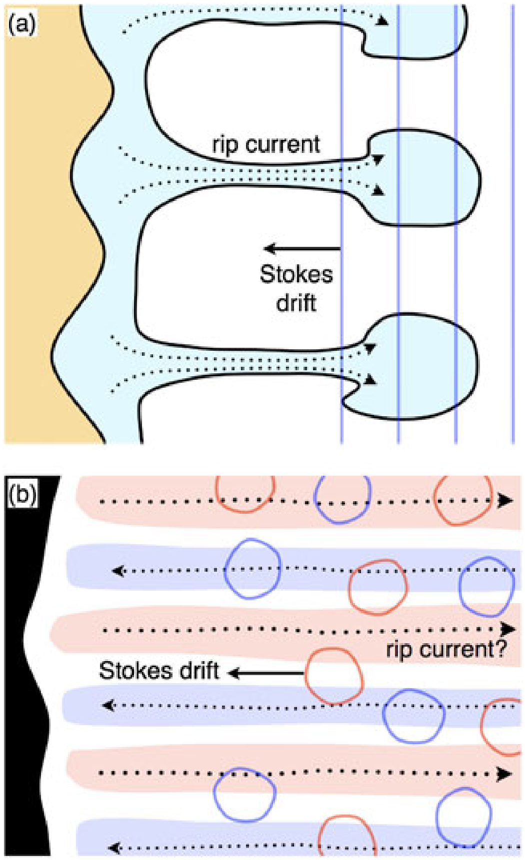

Figure 1. (a) Schematic of classical rip currents. Surface waves approaching a beach (blue lines) transport fluid onshore through their Stokes drift. Rip currents are required to return the fluid offshore and, due to complex dynamical balances, take the form of intense, narrow jets. (b) Schematic illustrating the origin of Rossby rip currents. Westward propagating Rossby waves and eddies (red and blue rings) transport fluid onshore, requiring Rossby rip currents to return the water offshore, possibly in form of quasi-zonal jets.

the fluid converging on the western boundary layers may be returned to the open ocean through *Rossby rip currents*.

[10] Consider a linear model of long Rossby waves in a reduced-gravity ocean:

$$u' = -\frac{g_p}{f}\frac{\partial h'}{\partial y}, \quad v' = \frac{g_p}{f}\frac{\partial h'}{\partial x}, \tag{3}$$

$$\frac{\partial h'}{\partial t} + h_0\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0, \tag{4}$$

where $f(y)$ is the Coriolis parameter and the remaining symbols are as previously defined. It follows that all anomalies propagate westward,

$$\frac{\partial h'}{\partial t} - c_r\frac{\partial h'}{\partial x} = 0, \tag{5}$$

at the long Rossby wave speed

$$c_r = \frac{\beta g_p h_0}{f^2}. \tag{6}$$

[11] We can calculate the integral bolus, or Stokes drift, velocity associated with a patch of eddy layer thickness variance, $\overline{h'^2}$, as follows:

$$\int \frac{\overline{h'u'}}{h_0}\,\mathrm{d}y = -\int \frac{\overline{h'^2}}{2h_0^2}c_r\,\mathrm{d}y, \tag{7}$$

where we have integrated by parts and assumed $\overline{h'^2}$ vanishes at the limits of integration. This expression is in exactly the same form as (1) for the Stokes drift associated with linear surface waves, except that the mean ocean depth is replaced by the mean upper layer thickness, the surface wave speed is replaced by the long Rossby wave speed, and (7) holds only in the weak integral sense.

[12] More generally, we can decompose the bolus or Stokes drift transport into rotational and divergent components:

$$\overline{h'\mathbf{u}'} = \mathbf{k}\times\nabla\left(\frac{g_p\overline{h'^2}}{2f}\right) - \frac{\overline{h'^2}}{2h_0}c_r\mathbf{i}, \tag{8}$$

where $\mathbf{i}$ is a zonal unit vector. The first term describes a rotational transport with vanishing divergence. (Strictly, this first term is purely rotational only if $\overline{h'^2}$ vanishes along the boundary—the sea surface height variance, proportional to $\overline{h'^2}$, is found to be small at the western boundary [*Kanzow et al.*, 2009]; while small divergences and convergences may remain along the western boundary, requiring alongshore currents, these have no impact on the net zonal transport into the western boundary layers and hence on the strength of the zonal rip currents.) In contrast, the second term is largely divergent and represents the westward Stokes drift in the integral expression (7).

3. Numerical Calculation

[13] We now illustrate the Rossby rip current mechanism through a numerical calculation with a nonlinear reduced-gravity ocean model. The model is discretized on a C-grid:

$$\frac{\partial u}{\partial t} - \left(f + \overline{\zeta}^{y}\right)\overline{v}^{xy} + \frac{\partial B}{\partial x} = A\nabla^2 u, \tag{9}$$

$$\frac{\partial v}{\partial t} + \left(f + \overline{\zeta}^{x}\right)\overline{u}^{xy} + \frac{\partial B}{\partial y} = A\nabla^2 v, \tag{10}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left(\overline{h}^{x}u\right) + \frac{\partial}{\partial y}\left(\overline{h}^{y}v\right) = -w_{Ek}, \tag{11}$$

where $\zeta = \partial v/\partial x - \partial u/\partial y$ is the relative vorticity, $B = g_p h + \left(\overline{u}^{x2} + \overline{v}^{y2}\right)/2$ is the Bernoulli potential, w_{Ek} is the Ekman upwelling velocity, and overbars indicate averages between adjacent grid points in the x and y directions. The time steps are discretized using a third-order Adams-Bashforth scheme with a time step of 2×10^{-5} year $\approx$ 631 s. The grid spacing is 10 km.

[14] Here we show the results from just one calculation in a square basin, extending from (0,0) to (L,L) where L = 4000 km, on the β plane, $f = f_0 + \beta y$ where $f_0 = 0.2 \times 10^{-4}$ s^{-1} and $\beta = 2 \times 10^{-11}$ m^{-1} s^{-1}. The Ekman upwelling vanishes except in the easternmost part of the domain ($x \geq 7L/8$), where

$$w_{Ek} = w_0\left(1 - \cos^{20}\frac{\pi y}{L}\right)\left(1 - \cos^{20}\frac{8\pi x}{L}\right)\sin\omega t, \tag{12}$$

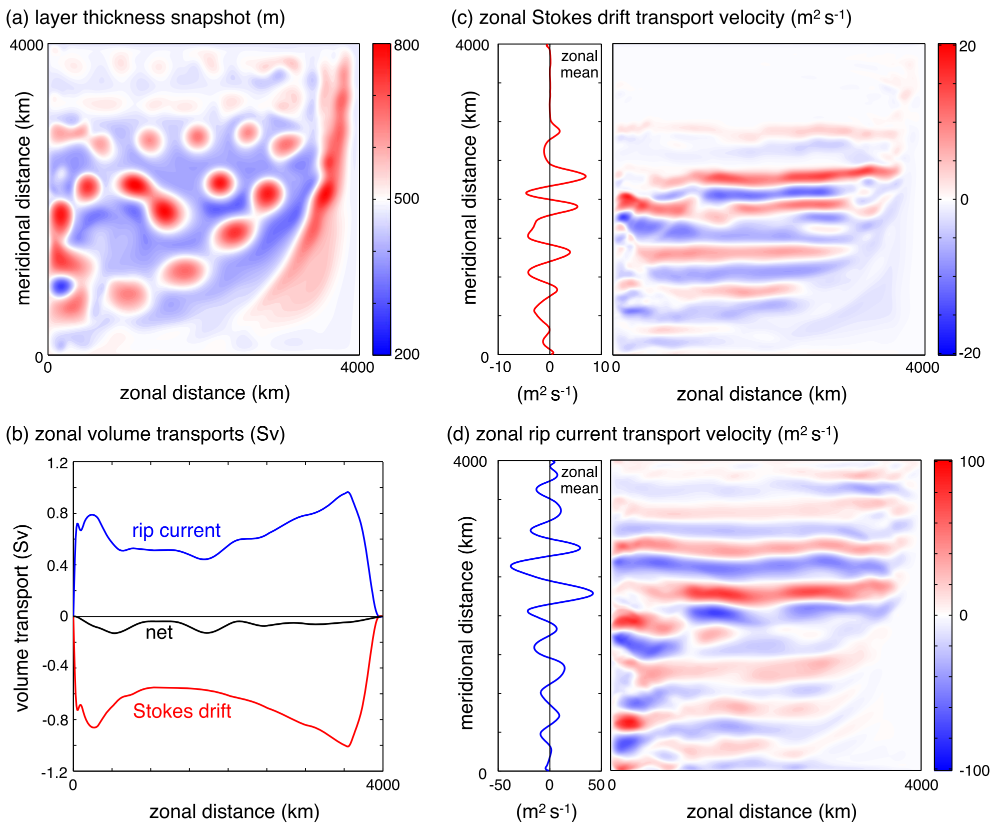

Figure 2. Results of the reduced-gravity numerical model calculation. (a) Instantaneous layer thickness (m) after 40 years. (b) Net zonal volume transports (Sv) by the Stokes drift, $\overline{h'u'}$, and Eulerian-mean flow, $\overline{h}\,\overline{u}$, as a function of zonal distance, averaged between years 21 and 40. (c) Zonal Stokes drift transport velocity, $\overline{h'u'}$, averaged over years 39–40, and its zonal mean. (d) Zonal Eulerian-mean transport velocity, $\overline{h}\,\overline{u}$, averaged over years 39–40, and its zonal mean.

acting as a Rossby wave maker. The forcing period is 1 year and amplitude $w_0 = 2 \times 10^{-5}$ m s^{-1}, chosen to give levels of eddy activity comparable to those observed adjacent to the western boundaries of the ocean.

[15] Plotted in Figure 2a is a snapshot of the layer thickness, h, after 40 years of integration. The Rossby waves are generated at the eastern margin of the domain and propagate westward, crossing the basin in about 0.5 year at the southern boundary and 12.5 years at the northern boundary. Consistent with previous studies [*LaCasce and Pedlosky*, 2004; *O'Reilly et al.*, 2012], the Rossby waves are unstable and break up into zonal jet-like features; the anticyclonic vorticity anomalies, in turn, break up into coherent vortices. While previous work has focused on the instability of two- and multi-layer Rossby waves [e.g., *Kim*, 1978; *Jones*, 1979], a generic argument for Rossby wave instability that applies in the reduced-gravity limit is given by *Pedlosky* [1987, section 7.17], invoking a necessary condition for instability due to *Arnold* [1965].

[16] In Figure 2b, we plot the net zonal volume transport by the Stokes drift, $\overline{h'u'}$, the Eulerian-mean flow, $\overline{h}\,\overline{u}$, and their sum, as a function of zonal distance across the channel, averaged over years 21–40. The Stokes drift is uniformly westward, of magnitude 0.6–0.9 Sv, compensated to leading order by an opposite Eulerian-mean transport. The residual is small, but finite, since the layer thickness is able to fluctuate on the time scales of a few Rossby waves crossing time scales. The sign and magnitude of the residual fluctuate depending on the precise averaging period used, but the general result holds even on much shorter time scales of a couple of years.

[17] Finally, in Figures 2c and 2d, we plot the time-mean zonal Stokes drift transport velocity, $\overline{h'u'}$, and Eulerian-mean transport velocity, $\overline{h}\,\overline{u}$, over a 2 year window (years 39–40), as well as the zonal averages of each. Both the Stokes drift and rip current assume the form of alternating quasi-zonal jets. In this particular experiment, the jets persist over far longer averaging periods (e.g., 20 years), although the precise location and magnitude of the jets are sensitive to the timing and length of the averaging window. The local values of Stokes drift are almost an order of magnitude smaller than the rip currents and more confined to lower latitudes. In both cases, the net zonal transports shown in Figure 2b are small residuals of alternating westward and eastward flows, although the rip currents are more strongly biased westward at low latitudes.

[18] Thus, one possible physical interpretation of the quasi-zonal jets observed in the ocean when averaging zonal velocity anomalies over suitable periods [*Maximenko et al.*, 2005, 2008] is that they represent the rip currents required to balance the westward Stokes drift associated with westward propagating Rossby waves and eddies. However, we have no grounds for believing that the quasi-zonal jets are a consequence of the westward Stokes drift but merely that in practice both the Stokes drift and resultant rip currents assume a jet-like structure due to the generic processes

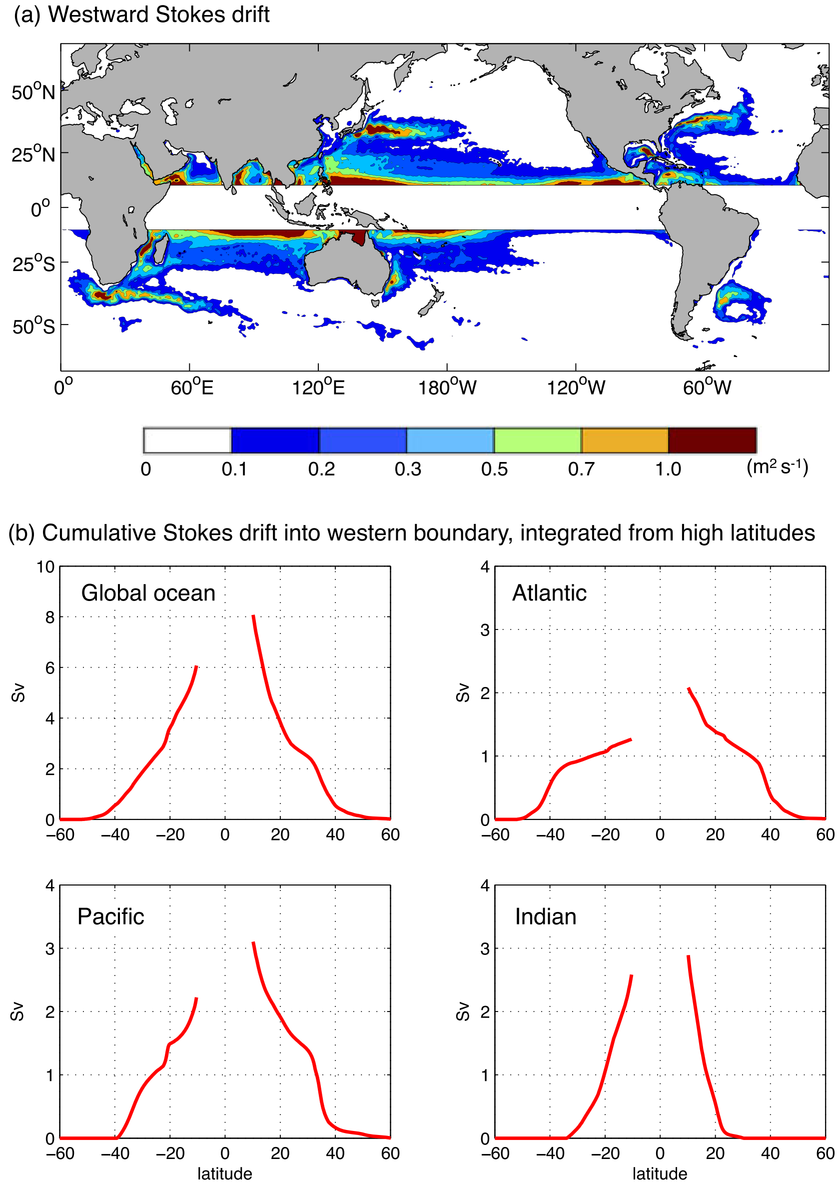

Figure 3. (a) Estimate of the rectified westward Stokes drift transport velocity, $(\overline{h'^2}/2h_0)c_r$, in (8), diagnosed from AVISO sea surface height data. Regions equatorward of 10° latitude are excluded. (b) Cumulative Stokes drift into the western boundary (Sv) integrated from a latitude of 60° in each hemispheric basin or its poleward margin where appropriate.

that favor jet formation. This is analogous to classical rip currents, where the reasons that the rip currents take the form of narrow jets are distinct from the need for a mean flow to compensate for the Stokes drift of the surface waves [see *Dalrymple et al.*, 2011].

4. Stokes Drift From Satellite Data

[19] Finally, we estimate the magnitude of the westward Stokes drift in the first baroclinic mode from satellite observations of sea surface height variability. Since the first term in (8) represents a recirculation, we focus on the second term in (8), the rectified westward Stokes drift (Figure 3a),

$$\frac{\overline{h'^2}}{2h_0}c_r = \frac{\beta g^2}{2g_p f^2}\overline{\eta'^2}. \qquad (13)$$

Here η' is the sea surface height anomaly, which we assume is dominated by the first baroclinic mode. The methodology is as described in *Zhai et al.* [2010], employing the AVISO merged altimetric data set (1992–2011) to estimate $\overline{\eta'^2}$ with $g_p = 1.5 \times 10^{-2}$ m s^{-2}.

[20] The largest values of the Stokes drift are found in the western parts of the basin and within two latitude bands. The first is the separated midlatitude boundary currents, where the eddy field is most energetic, and the second is at lower latitudes due to the inverse relation of the Stokes drift with the square of the Coriolis parameter. While not conclusive, a superficial comparison with Figure 1b of *Maximenko et al.* [2005] suggests some correspondence of regions of largest Stokes drift and largest quasi-zonal jets; in particular, the absence of significant jet-like features in the low-latitude South Atlantic is striking.

[21] The one exception is in the Southern Ocean, where the Stokes drift is modest yet jet-like structures are prevalent, embedded within the core of the Antarctic Circumpolar Current. Consistent with the previous discussion, various processes may lead to jet formation in the Southern Ocean [e.g., *Galperin et al.*, 2004; *Berloff et al.*, 2009a, 2009b]. However, in contrast to the ocean basins, the absence of continental barriers removes the requirement for rectified rip currents to close the volume budget in the Southern Ocean.

[22] To estimate the magnitude of the Stokes drift in each hemispheric basin, in Figure 3b, we present the cumulative Stokes drift into the western boundary region integrated between the poleward extreme of the basin (or 60°N/S) and the plotted latitude (note that the Agulhas region in the Indian Ocean is excluded since there is no western boundary). The net Stokes drift is around 2 Sv in each hemispheric basin (smaller in the South Atlantic) and about 14 Sv globally. However, we caution that the precise values are acutely sensitive to the low-latitude limit of the integration and should be viewed as having large uncertainty. Nevertheless, these estimates are useful in defining the magnitude of the Stokes drift which, while small, is nonnegligible in the context of the global ocean circulation.

5. Concluding Remarks

[23] We have demonstrated, through a combination of linear theory, numerical calculations, and analysis of satellite sea surface height data, that Rossby waves and eddies transport energy and fluid westward, the latter through the bolus transport or Stokes drift. While the energy is mostly dissipated at the western boundary [*Zhai et al.*, 2010], the fluid may be returned eastward through Eulerian-mean currents, which we term *Rossby rip currents*. Integrated over the global ocean, the offshore volume transport of these Rossby rip currents is of the order 10 Sv.

[24] The numerical model calculations suggest that both the rip currents and the Stokes drift can assume the form of alternating zonal jets, with a rectified volume transport. This raises the intriguing possibility that the quasi-zonal jets recently observed in the ocean through analysis of altimetric data [e.g., *Maximenko et al.*, 2005, 2008], and also output of eddy-permitting general circulation models [e.g., *Richards et al.*, 2008], are the manifestation of rip currents in the ocean. However, just as the fluid dynamics responsible for shaping classical rip currents are complex, involving wave radiation stresses and bathymetric geometry among numerous other processes [*Dalrymple et al.*, 2011], so it seems likely the quasi-zonal jets observed in the ocean are caused through a myriad of processes [see, e.g., *Galperin et al.*, 2004; *LaCasce and Pedlosky*, 2004; *Hristova et al.*, 2008; *Berloff et al.*, 2009a, 2009b; *Kamenkovich et al.*, 2009; *Afanasyev et al.*, 2011; *O'Reilly et al.*, 2012]. While the Rossby rip currents may well assume the form of banded zonal jets, for numerous reasons unrelated to the rip currents themselves, the more fundamental result seems the need for their existence in order to balance the ubiquitous westward Stokes drift of Rossby waves and eddies.

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References

Afanasyev, Y., S. OLeary, P. Rhines, and E. Lindahl (2011), On the origin of jets in the ocean, *Geophys. Astrophys. Fluid Dyn.*, *106*, 113–137.

Arnold, V. I. (1965), Conditions for nonlinear stability of stationary plane curvilinear flows of an ideal fluid, *Dokl. Akad. Nauk SSSR*, *162*, 975–978.

Berloff, P., I. Kamenkovich, and J. Pedlosky (2009a), A model of multiple zonal jets in the oceans: Dynamical and kinematical analysis, *J. Phys. Oceanogr.*, *39*, 2711–2734.

Berloff, P., I. Kamenkovich, and J. Pedlosky (2009b), A mechanism of formation of multiple zonal jets in the oceans, *J. Fluid Mech.*, *628*, 395–425.

Chelton, D. B., M. G. Schlax, R. M. Samelson, and R. A. de Szoeke (2007), Global observations of large oceanic eddies, *Geophys. Res. Lett.*, *34*, L15606, doi:10.1029/ 2007GL030812.

Dalrymple, R. A., J. H. MacMahan, J. H. M. Reniers, and V. Nelko (2011), Rip currents, *Annu. Rev. Fluid Mech.*, *43*, 551–581.

Galperin, B., H. Nakano, H.-P. Huang, and S. Sukoriansky (2004), The ubiquitous zonal jets in the atmospheres of giant planets and Earth's oceans, *Geophys. Res. Lett.*, *31*, L13303, doi:10.1029/2004GL019691.

Gent, P. R., J. Willebrand, T. J. McDougall, and J. C. McWilliams (1995), Parameterizing eddy-induced tracer transports in ocean circulation models, *J. Phys. Oceanogr.*, *25*, 463–474.

Hristova, H., J. Pedlosky, and M. Spall (2008), Radiating instability of a meridional boundary current, *J. Phys. Oceanogr.*, *38*, 2294–2307.

Jones, S. (1979), Rossby wave interactions and instabilities in a rotating, two-layer fluid on a beta-plane, *Geophys. Astrophys. Fluid Dyn.*, *12*, 1–33.

Kamenkovich, I., P. Berloff, and J. Pedlosky (2009), Role of eddy forcing in the dynamics of multiple zonal jets in a model of the North Atlantic, *J. Phys. Oceanogr.*, *39*, 1361–1379.

Kanzow, T., H. L. Johnson, D. P. Marshall, S. A. Cunningham, J. J.-M. Hirschi, A. Mujahid, and H. L. Bryden (2009), Basinwide integrated volume transports in an eddy-filled ocean, *J. Phys. Oceanogr.*, *39*, 3091–3110.

Kim, K. (1978), Instability of baroclinic Rossby waves; energetics in a two-layer ocean, *Deep Sea Res.*, *25*, 795–814.

LaCasce, J., and J. Pedlosky (2004), The instability of Rossby basin modes and the oceanic eddy field, *J. Phys. Oceanogr.*, *34*, 2027–2041.

Lee, M. M., D. P. Marshall, and R. G. Williams (1997), On the eddy transfer of tracers: Advective or diffusive? *J. Mar. Res.*, *55*, 483–505.

Maximenko, N. A., B. Bang, and H. Sasaki (2005), Observational evidence of alternating zonal jets in the world ocean, *Geophys. Res. Lett.*, *32*, L12607, doi:10.1029/2005GL022728.

Maximenko, N. A., O. V. Melnichenko, P. P. Niiler, and H. Sasaki (2008), Stationary mesoscale jet-like features in the ocean, *Geophys. Res. Lett.*, *35*, L08603, doi:10.1029/2008GL033267.

O'Reilly, C. H., A. Czaja, and J. H. LaCasce (2012), The emergence of zonal ocean jets under large-scale stochastic wind forcing, *Geophys. Res. Lett.*, *39*, L14605, doi:10.1029/2007GL030379.

Pedlosky, J. (1987), *Geophysical Fluid Dynamics*, Springer-Verlag, New York.

Richards, K. J., H. Sasaki, and F. O. Bryan (2008), Jets and waves in the Pacific Ocean, in *High Resolution Numerical Modelling of the Atmosphere and Ocean*, edited by W. Ohfuchi, and K. Hamilton, pp. 182–196, Springer, New York.

Zhai, X., H. L. Johnson, and D. P. Marshall (2010), Significant sink of ocean-eddy energy near western boundaries, *Nat. Geosci.*, *3*, 608–612.